Ideas for Situations Conference

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1 Transforming expressions

1.1 Prompt

Middle school teachers taking a course in algebra are given the

- The expression

  $$0.6\left(\frac{t_1+t_2+t_3}{3}\right)$$

  is the contribution to a student's final score from three test scores. What is a different way of writing this? Which way should a student use in order to

  - calculate the total test contribution to their final grade
  - calculate the effect of getting 10 more points on test 2

They generate various responses:

$$0.6\left(\frac{t_1+t_2+t_3}{3}\right), 0.2t_1+0.2t_2+0.2t_3, \frac{t_1}{5}+\frac{t_2}{5}+\frac{t_3}{5}, \ldots$$

and then the following conversation ensues:

Student A: I wrote (2) because I thought that the original expression said the average of the 3 tests was worth 60%, so each test was worth 20%. But I'm not sure it is right.

Student B: (1) and (2) are obviously the same!

Student A: How you can see that just by looking at them?

Student B: You just move the 3 over so it's dividing the 0.6, which gives you 0.2, then distributed the 0.2.

Instructor: How do you know you can move the 3 over? What rule says you can do that?

Student B: Isn't it because you only have division and multiplication, so it's the commutative law?

Instructor: But division isn't commutative.

Student C: But you can write division as multiplication. Just write it as multiplication by 1/3.

Student A: Oh yeah! [Discussion shifts to associative law.]

1.2 Mathematical foci

Mathematical Focus 1

The rules of arithmetic determine what transformations on an algebraic expression will produce an equivalent expression.

Mathematical Focus 2

In using the rules of arithmetic to justify common transformations of algebraic fractions such as "canceling" or "moving the denominator" we need to make explicit the translation between fraction notation and the notation of field operations.

2 Solving a quadratic equation by factoring

2.1 Prompt

A class of prospective high school teachers is considering the solution of a quadratic equation: If

$$x^2 - 3x - 4 = 0 \tag{1}$$

then

$$(x-4)(x+1) = 0 \tag{2}$$

so

$$x - 4 = 0 \quad \text{or} \quad x + 1 = 0, \quad \text{so} \quad x = 4 \quad \text{or} \quad x = -1. \tag{3}$$

The instructors asks how the step from (2) to (3) is justified. The following conversation ensues:

Student A: You can say 0 times A, where A is any real number, is equal to 0, so you can set either of them equal to zero to make the entire let half equal to 0.

Student B: And that's the only way to get 0, you can't get it any other way.

Instructor: But you just said two different things, you [Student A] said that 0 times any number is 0, and you [Student B] siad that's the only way to get 0. Are those the same thing?

Student B: They both apply to this equation. What you have the right hand side is 0, so how are you going to get 0, and we know that A times 0 is 0.

Instructor: OK, let's just figure out and make sure we know what the logical statement is. Give me an if-then statement that is the one we are using here.

Student C: If $AB = 0$ then $A = 0$ or $B = 0$.

Instructor: Is that what Student A said when he justified his step? [Pointing to Student A.] Say again what you said.

Student A: I said if you have the two factors at least one of them needs to be 0 for this equation to be 0.

Student D: He identified that there's a property that if you multiply any number by 0, you get 0.

Instructor: Is that the same as this? [Pointing to board.]
Student D: Not precisely.
Instructor: Can you say what you just said in an if-then statement?
[Discussion continues. I have video of this.]

2.2 Mathematical Foci

2.2.1 Mathematical Focus 1

Solving equations is a process of logical deduction in which each equation is a statement deduced from the previous equation.

2.2.2 Mathematical Focus 2

In solving quadratic equations by factoring, we use the fact that $AB = 0$ *if and only if* $A = 0$ *or* $B = 0$*. One direction of this implication is used to conclude that each of the factors must be zero, the other is used to verify that zeros of each factor are zeros of the quadratic expression.*